Analysis of a Work Exchanger Network

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In the last few decades, heat exchanger networks have been widely adopted to recover thermal energy. Recently, mass exchanger networks have gained attention for recovering useful materials. By analogy, another class of exchanger networks, termed work exchanger networks (WEN's), is conceivable for recovering mechanical energy. In the present work, the concept of the WEN is introduced. The WEN consists of a number of work exchangers, each of which serves to exchange mechanical energy between a high-pressure stream and a low-pressure stream. By applying the basic principles of energy balance and thermodynamics laws, the necessary and sufficient conditions are derived to identify feasible stream matching in work exchangers. A comprehensive analysis of stream match mode gives rise to various heuristic rules useful for WEN synthesis. Our focus is on the analysis, rather than the synthesis, of the WEN. This work lays a foundation to the development of a systematic WEN synthesis methodology.

1. Introduction

During the last 2 decades, the design of heat exchanger networks (HEN's) by the pinch technology has led to the recovery of large amounts of thermal energy in the process and allied industries (Linnhoff et al., 1982; Lu and Motard, 1985). Recently, El-Halwagi and Manousiouthakis (1989, 1990) introduced a novel concept of mass exchanger networks (MEN's). A MEN is capable of significantly recovering valuable materials and minimizing various hazardous wastes (El-Halwagi and Srinivas, 1992; Huang and Fan, 1992, 1993a, 1994; Edgar and Huang, 1994; Srinivas and El-Halwagi, 1994). More recently, the synthesis of a combined exchanger network was introduced for the simultaneous recovery of thermal energy and chemicals in chemical and petroleum refining processes (Huang, 1992; El-Halwagi et al., 1995; Huang and Edgar, 1995).

From the point of view of thermodynamics, the state variables in HEN's and MEN's are temperatures and concentrations, respectively. Needless to say, pressure is another important state variable for a variety of process systems requiring pressurization and depressurization operations. Some examples are ammonia synthesis, reverse osmosis, freezing purification based on high pressure inversion on the order of melting points, manufacture of phenol by the hydrolysis of chlorobenzene, and hydrogenation of oil and coal. In these processes, some feed streams need be pressurized up to 1000 kg/cm² from a very low pressure through four or more stages of compression, while some product streams need be depressurized from a very high pressure (e.g., 1000 kg/cm²) to a very low pressure after reactions or other unit operations. A practical pressurization/depressurization problem in an ammonia plant is illustrated in Table 1 (Huang and Fan, 1993b). In this problem, the pressures of all five streams must be either increased or decreased in order to meet process specifications. The simplest solution is to install a compressor or pump if a stream is to be pressurized and an expander or a turbine if a stream is to be depressurized. This implies considerable capital investment and operating cost. An energy balance calculation shows that the mechanical energy available from the two high pressure streams is 58.212 kg-cm/s, and the mechanical energy required by the three low pressure streams is 63.135 kg-cm/s. If the available mechanical energy in the high-pressure streams is sufficiently utilized to pressurize the low pressure streams, then the operating cost can be greatly reduced; this is a very significant economic implication. This suggests that a process system should be devised to perform work exchange.

In reality, a generalization of the notions of HEN and MEN synthesis can give rise to the notion of work exchanger network (WEN) synthesis. A WEN is a process system containing two or more work transfer units (WTU's) which exchange work among process streams. Such a transfer unit, named a flow work exchanger (WE), was originally conceived by Cheng et al. (1967); it was eventually fabricated and successfully placed in operation for a reverse osmosis process for desalination; see Figure 1 (Cheng et al., 1968). The WE was also reported to be useful for another desalination process which was based on the inversion of melting points due to applied pressure: this application resulted in significant economic incentives (Cheng and Cheng, 1967).

2. Work Exchanger

Whenever a high-pressure fluid stream, or simply stream HP, is depressurized in such a manner that throttling occurs, a potential exists for mechanical energy recovery. On the other hand, pressurizing a low-

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Table 1. Design Data of the WEN Synthesis Problem

<table>
<thead>
<tr>
<th>stream no.</th>
<th>source pressure $P_s$ (kg/cm²)</th>
<th>target pressure $P_t$ (kg/cm²)</th>
<th>volumetric flowrate $V$ (cm³/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP1</td>
<td>58.35</td>
<td>22.50</td>
<td>990.49</td>
</tr>
<tr>
<td>HP2</td>
<td>31.64</td>
<td>1.90</td>
<td>763.37</td>
</tr>
<tr>
<td>LP1</td>
<td>2.39</td>
<td>58.35</td>
<td>725.52</td>
</tr>
<tr>
<td>LP2</td>
<td>1.05</td>
<td>22.50</td>
<td>504.71</td>
</tr>
<tr>
<td>LP3</td>
<td>17.58</td>
<td>31.64</td>
<td>832.77</td>
</tr>
</tbody>
</table>
pressure fluid stream, or simply stream LP, always requires mechanical energy. Obviously, a match of the two streams in a WE will transfer mechanical energy as shaft work from stream HP to stream LP (Cheng et al., 1967).

It is assumed that stream LP is pressurized from source pressure $P_{LP}^s$ to target pressure $P_{LP}^t$ and that stream HP is depressurized from source pressure $P_{HP}^s$ to target pressure $P_{HP}^t$. Accordingly, the reversible shaft work of each stream can be expressed as follows (Kyle, 1992):

$$(-W_{LP})_t = \int_{P_{LP}^s}^{P_{LP}^t} P \, dV = \int_{V_{LP}^s}^{V_{LP}^t} \left( P_{LP}^s V_{LP}^t - P_{LP}^t V_{LP}^s \right) dV$$  \hspace{1cm} (1)

$$(+W_{HP})_t = \int_{P_{HP}^s}^{P_{HP}^t} P \, dV = \int_{V_{HP}^s}^{V_{HP}^t} \left( P_{HP}^s V_{HP}^t - P_{HP}^t V_{HP}^s \right) dV$$  \hspace{1cm} (2)

Each expression contains two terms: one being the difference in the flow work under the high and low pressures and the other being the shaft work for a corresponding nonflow process. The difference of these two terms gives the shaft work for a flow process. Note that the greater the difference in the flow work terms, $\Delta(PV)$s, the greater the shaft work for the reversible pressurization and depressurization.

A WE containing two displacement vessels is sketched in Figure 2 (Cheng et al., 1967). The work exchange process involves the following four consecutive steps.

(a) Substantially nonflow depressurization. Displacement vessel 1 is filled with the high-pressure product at $P_{HP}^s$. By closing valve $V_1$ and opening valve $V_5$, the content in the vessel is depressurized, and some product fluid flows out of the vessel through valve $V_6$ in the amount corresponding to the volume expansion due to depressurization. This operation takes a very short time. Valves $V_1$ and $V_2$ are in the closed position during this operation.

(b) Low-pressure displacement. When the pressure on the right-hand side of the piston (solid partitioner) of vessel 1 drops below $P_{LP}^t$, valve $V_2$ opens and the low-pressure feed flows into the left-hand side of the piston through valve $V_2$ and the depressurized product flows out of the vessel through valve $V_6$. The piston moves from the left end to the right end, and valves $V_1$ and $V_5$ are kept closed. At the end of this operation, the vessel is filled with the low-pressure feed on the left-hand side of the piston.
Filled with high-pressure product at hand end. At the end of this operation the vessel is in vessel 2 moves from the right-hand end to the left-hand side of the piston of the vessel to pressurize the content. This operation takes a very short time because only a small amount of fluid sufficient to compensate for the volume shrinkage has to be introduced. During this operation, valves V3 and V4 are in the closed position.

(d) High-pressure displacement. When the pressure on the left-hand side of the piston in the vessel exceeds $P_{LP}$, valve V3 opens, and the high-pressure product flows continuously into the vessel through valve V7 and the pressurized feed fluid is displaced through valve V3 into the high-pressure processing system. The piston in vessel 2 moves from the right-hand end to the left-hand end. At the end of this operation the vessel is filled with high-pressure product at $P_{HP}$. Then, the cycle (or operation) returns to step (a) and starts over again.

The information on pressure change and valve position is included in Figure 3. It has been shown unequivocally that the WE is applicable to any high-pressure processing system in which one stream should be pressurized before processing and then be depressurized after processing (Fan et al., 1968). It is also desirable to recover mechanical energy in the process where different pressures are required in various units (Huang and Fan, 1993).

### 3. From Heat and Mass Exchanger Networks to a Work Exchanger Network

Thermodynamically, any object can be regarded as possessing potential to undergo a spontaneous change or transformation when it is not in equilibrium with the environment. This potential transfers from one object to another or to the environment to drive the system toward the equilibrium (Kyle, 1992). In a heat exchanger (HE) or mass exchanger (ME), the potential of a hot (or rich) stream is higher than that of a cold (or lean) stream in terms of thermal energy (or of the concentration of a certain key chemical species or component). By the same token, in a WE, the potential of stream HP is higher than that of stream LP in terms of mechanical energy. Each of these types of exchangers transfers a type of potential because of the existence of a driving force. In a HE, the driving force is the temperature difference which permits a heat flow; in a ME, it is the concentration difference which allows a mass flow; and in a WE, the driving force is the pressure difference which causes a work flow.

#### Similarities among the Three Types of Exchanger Networks.

A process consisting of two or more HE's (or ME's) is termed a HEN (or MEN). In the HEN (or MEN), the state variable characterizing it is temperature (or concentration); the process occurring in it is heat (or mass) transfer; and the function performed by it is heat (or mass) exchange. By analogy, it is natural and logical to introduce the concept of a work exchanger network (WEN). A WEN should consist of two or more WE's. In the WEN, the state variable is pressure; the process occurring in it is mechanical energy transfer; and the function performed by it is work exchange.

The notion of WEN synthesis is a natural extension of those of HEN and MEN syntheses. Let us first examine the equilibrium relations and operating line expressions inherent in these exchanger networks.

In a HEN, an equilibrium relation exists between a pair of hot stream $H_i$ and cold stream $C_j$ which perform heat change in a HE. Although it has seldom been mentioned in the open literature, the equilibrium relation can be simply written as

$$T_{Hi} = T_{Cj}$$

where $T_{Hi}$ is the temperature of hot stream $H_i$, and $T_{Cj}$ is that of cold stream $C_j$.

In a MEN, the equilibrium solubility relation for a key component between rich stream $R_i$ and lean stream $L_j$ through a ME is often approximated by the following linear relation.

$$C_{Pi} = aC_{Pj} + b$$

where $C_{Pi}$ is the composition or concentration of the key component, $p$, in rich stream $R_i$; $C_{Pj}$ is that in lean stream $L_j$; and $a$ and $b$ are constants.

By comparing the forms of eqs 3 and 4, a general equilibrium relation can be derived as

$$E_{HS_i} = aE_{LS_j} + b$$

where $E_{HS_i}$ and $E_{LS_j}$ are the state variable of high-potential stream HS and that of low-potential stream LS, respectively, and $a$ and $b$ are constants. This general equilibrium relation can certainly be applied to the conceived WEN where pressure is the state variable. For the WEN, however, $a$ and $b$ in eq 5 should be equal to 1 and 0, respectively, i.e.

$$P_{HP} = P_{LP}$$

where $P_{HP}$ and $P_{LP}$ are the pressures of high-pressure stream HP and low-pressure stream LP, respectively.

Similarities between the operating lines of a HEN and a MEN can be extended to a WEN. The equation describing an operating line in any HE of a HEN is as follows:

$$M_{C_{Pi}}(T_{Hi}^t - T_{Hi}^s) = -M_{C_{Pj}}(T_{Cj}^t - T_{Cj}^s)$$

or it can be simply written in terms of the heat load, $Q$, for each stream as

$$Q_{Hi} = -Q_{Cj}$$

For a MEN, the equation describing an operating line in any ME has the same form as that in any HE of a...
Table 2. Comparison of Various Aspects of Exchanger Network Syntheses

<table>
<thead>
<tr>
<th>type of exchanger network</th>
<th>heat exchanger network (HEN)</th>
<th>mass exchanger network (MEN)</th>
<th>work exchanger network (WEN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>process</td>
<td>heat transfer</td>
<td>mass transfer</td>
<td>mechanical energy transfer</td>
</tr>
<tr>
<td>macroscopic function</td>
<td>heat exchange</td>
<td>mass exchange</td>
<td>work exchange</td>
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<tr>
<td>equipment</td>
<td>heat exchanger (HE)</td>
<td>mass exchanger (ME)</td>
<td>work exchanger (WE)</td>
</tr>
<tr>
<td></td>
<td>heater (HU)</td>
<td>enricher (EU)</td>
<td>compressor (PU)</td>
</tr>
<tr>
<td></td>
<td>cooler (CU)</td>
<td>dilutor (DU)</td>
<td>expander (EU)</td>
</tr>
<tr>
<td>flow</td>
<td>heat flow</td>
<td>mass flow</td>
<td>work flow</td>
</tr>
<tr>
<td>process stream</td>
<td>hot stream $H_i$</td>
<td>rich stream $R_i$</td>
<td>high-pressure stream $H_{P_i}$</td>
</tr>
<tr>
<td></td>
<td>cold stream $C_i$</td>
<td>lean stream $L_i$</td>
<td>low-pressure stream $L_{P_i}$</td>
</tr>
<tr>
<td>utility stream</td>
<td>steam $H_{US}$</td>
<td>enriching stream $R_{US}$</td>
<td>high-pressure external stream $H_{P_{US}}$</td>
</tr>
<tr>
<td></td>
<td>cooling water $C_{US}$</td>
<td>diluting stream $L_{US}$</td>
<td>low-pressure external stream $L_{P_{US}}$</td>
</tr>
<tr>
<td>state variable</td>
<td>temp $T$</td>
<td>conc $C$</td>
<td>pressure $P$</td>
</tr>
<tr>
<td>capital cost (approximation)</td>
<td>no. of HE's, HU's, and CU's</td>
<td>no. of ME's, EU's, and DU's</td>
<td>no. of WE's, PU's, and EU's</td>
</tr>
<tr>
<td>criterion</td>
<td>consumption of thermal energy</td>
<td>consumption of mass separating agents</td>
<td></td>
</tr>
<tr>
<td>equilibrium relation</td>
<td>MER</td>
<td>MMSAR</td>
<td>MWR</td>
</tr>
<tr>
<td>load</td>
<td>$T_H = T_C$</td>
<td>$C_R = aC_b + b$</td>
<td>$P_{HP} = P_{SL}$</td>
</tr>
<tr>
<td>operating line</td>
<td>heat load $Q = MC_p(T_H^t - T_S^t)$</td>
<td>mass load $M_P = M(C_R^t - C_S^t)$</td>
<td>work load $W = V(P_{SL}^t - P_S^t)$</td>
</tr>
<tr>
<td></td>
<td>$Q_H = Q_C$</td>
<td>$M_P = M_R$</td>
<td>$W_{HP} = W_{LP}$</td>
</tr>
</tbody>
</table>

HEN, i.e.,

$$M_{R_i}(C_{P_{H_1}} - C_{S_{H_1}}) = -M_{L_j}(C_{P_{L_1}} - C_{S_{L_1}})$$ (9)

or it can be simply written in terms of the mass load of the key component $p$, $M_{P_i}$, for each stream as

$$M_{P_{H_i}} = -M_{P_{L_j}}$$ (10)

By examining the forms of eqs 7–10, the operating line of any WE in a WEN can be analogously derived as follows:

$$V_{H_{P_i}}(P_{H_{P_i}}^t - P_{H_{P_i}}^s) = -V_{L_{P_j}}(P_{L_{P_j}}^t - P_{L_{P_j}}^s)$$ (11)

or simply in terms of the work load, $W$, for each stream as

$$W_{H_{P_i}} = -W_{L_{P_j}}$$ (12)

Other similarities among these three types of exchanger networks are further identified as summarized in Table 2.

Statement of WEN Synthesis. As expected, analogies between the operational characteristics of the HEN and MEN manifest themselves in various aspects of their syntheses, such as synthesis criteria, the identification of system limit, and representation schemes. Such analogies naturally extend themselves to WEN synthesis as alluded to earlier.

It is certainly desirable that a WEN be economically attractive, i.e., it is advantageous for a plant to install a WEN to recover mechanical energy. The task of synthesizing the WEN consisting of two types of streams can be stated as follows:

Given set $H_P$ of $N_{HP}$ high-pressure streams including $N_{HP_{P_{H_1}}}$ high-pressure process streams and $N_{HP_{P_{P_{H_1}}}}$ high-pressure utility streams, and set $L_P$ of $N_{LP}$ low-pressure streams including $N_{LP_{L_1}}$ low-pressure process streams and $N_{LP_{P_{L_1}}}$ low-pressure utility streams, i.e., given

$$H_P = \{H_{P_1}, H_{P_2}, \ldots, H_{P_{N_{HP_{P_{H_1}}}}}, H_{P_{(N_{HP_{P_{H_1}}}+1)}}\}$$ (13)

and

$$L_P = \{L_{P_1}, L_{P_2}, \ldots, L_{P_{N_{LP_{L_1}}}}, L_{P_{(N_{LP_{L_1}}}+1)}\}$$

synthesize a WEN performing flow work transfer from the set of high-pressure streams to the set of low-pressure streams at the lowest possible capital and operating costs.

For each stream $S_i$, i.e., either $H_{P_i}$ or $L_{P_j}$, in a WEN synthesis problem, the following information must be provided:

$$S_i = \{p_{i_1}^t, p_{i_1}^s, V_i\}$$

Moreover, five major assumptions are imposed on the synthesis: (a) any material that has sufficient fluidity to be handled by a pump, expander, turbine, or compressor is regarded as a fluid, whether it is a liquid, slurry, or paste, (b) no phase change is involved, (c) a WTU is a displacement-vessel-based unit, (d) the work lost by the high-pressure stream is completely received by the low-pressure stream in a WTU, and (e) the lost work due to leakage of fluid caused by the volume inefficiencies of valves and by the inefficiencies of pumps is negligible.

Difficulties in WEN Synthesis. As described in the preceding section, a WE is operated essentially in a batch mode. In this mode, four consecutive steps are repeated. During operation, the source pressure of the low-pressure stream, i.e., $P_{SL}$, must be greater than the target pressure of the high-pressure stream, i.e., $P_{HP}$.

In contrast, a HE or ME is operated in a continuous mode, where the source temperature (or concentration) of the cold (or lean) stream, i.e., $T_{C}^t$ (or $C_{C_{C}}^t$), is always greater than the target temperature (or concentration) of the hot (or rich) stream, i.e., $T_{R}^t$ (or $C_{R_{C}}^t$). This fundamental difference disallows the direct application of the widely used pinch technology.

Similar to the HEN and MEN syntheses, the available mechanical energy in a set of high-pressure streams cannot be entirely exploited to pressurize low-pressure streams in a WEN. For the example in Table 1, the application of the thermodynamics laws shows that the available mechanical energy from high-pressure streams is 58 212 kg·cm/s and it can only be partially recovered for pressurizing low-pressure streams. Due to the fundamental difference mentioned above, the computation of the mechanical energy actually usable for pres-
surization is different. This makes the synthesis of the WEN much more difficult than that of the HEN or MEN. The WEN synthesis may still involve three major phases, i.e., preanalysis, structure invention, and structure evolution. Nevertheless, all necessary precautions must be taken in all synthesis phases to circumvent the difficulties attributable to the fundamental difference between the WEN synthesis and the HEN (or MEN) synthesis. To do so, a problem representation scheme and stream matching strategies should be developed before a systematic synthesis procedure can be conceived.

4. P–W Diagram—A Problem Representation Scheme

For a HEN (or MEN) synthesis problem, a T–Q [or C–M_p] diagram serves as a problem representation scheme to identify the pinch point and the minimum requirement of energy (or mass separating agents). By analogy, a P–W diagram should be a desirable problem representation scheme for a WEN. This is due to the fact that, corresponding to state variable T (or C) in a HEN (or MEN), we have state variable P in a WEN, and corresponding to load variable Q (or M_p) in a HEN (or MEN), there exists load variable W in a WEN (Table 2).

Cheng et al. (1967) resorted to a P–V diagram to analyze work exchange between stream HP and stream LP as illustrated in Figure 4. In this figure, lines 2–3 and 7–8–9 represent nonflow depressurization; lines 3–4 and 9–10, low-pressure displacement; lines 4–1 and 10–6–6', nonflow pressurization; and lines 1–2 and 6–7, high-pressure displacement. The shaded area represents the work required for this process. This two-dimensional P–V diagram facilitates the visualization of operational steps involved in work exchange between two streams. Nevertheless, it is insufficient for the case involving three or more streams, which requires the synthesis of a WEN.

Figure 4. P–V diagram for a flow work exchanger (Cheng et al., 1967).

For WEN synthesis, the visualization of the work energy exchanged among all streams is more important than that of the operational steps. This is similar to HEN synthesis of visualizing the thermal energy transfer rather than the heat capacity flow rate or MEN synthesis of visualizing the mass transfer rather than the mass flow rate. The implication is that it is more desirable to resort to the P–W diagram than to the P–V diagram. The P–W diagram can be derived by adding one dimension to the P–V diagram, as illustrated in Figure 5. The resultant three-dimensional diagram is termed the P–V–W diagram. The work exchange process illustrated in Figure 4 is replotted in this diagram. The two P–V planes, with plane 1 on the back of plane 2, are connected by two intersecting lines: (i) line 3–1” representing the pressurization of stream LP from its source pressure P_{LP,t} to target pressure P_{LP,LP}, and (ii) line 6”–8 representing the depressurization of stream HP from its source pressure P_{HP,HP} to target pressure P_{HP,HP}. These two lines are projected into the P–W plane.

The P–W diagram in Figure 6 is generated from the P–V–W diagram. The intersecting lines projected in the P–W plane in Figure 5 are now replotted in Figure 6. These two lines are, in reality, two operating lines representing the work exchange process between two streams. Note that work exchange occurs only because of the existence of driving forces at both high-pressure ends and low-pressure ends of the pair of streams. The pressurization and depressurization processes occur successively and alternatively, rather than simultaneously, in the WE (Cheng et al., 1967). Hence, the pinch point is located at either the high-pressure or low-pressure end only, whenever the driving force reaches the minimum. The pressure difference between the two points at a given W other than these two end values...
should not be regarded as the driving force for transferring work energy. This is entirely different from the synthesis of a HEN or MEN where more than two streams are involved. Consequently, the conventional approach for identifying a pinch point is not directly applicable to WEN synthesis.

5. Necessary and Sufficient Conditions for Stream Matching

As illustrated in Figure 6, the transfer of mechanical energy between a high-pressure stream and a low-pressure stream is considerably different from the transfer of thermal energy (or mass) between a hot (or rich) stream and a cold (or lean) stream. Thus, the necessary and sufficient conditions should be established for stream matching in WEN synthesis.

(i) For any WTU in a WEN to function, the depressurization of stream HP, and vice versa; obviously,

\[
P_{HP}^s > P_{HP}^t \quad \text{for stream HP} \]
\[
P_{LP}^s < P_{LP}^t \quad \text{for stream LP} \quad (16)
\]

Furthermore, the work energy should not be exchanged between any pair of high-pressure streams; this is also the case between any pair of low-pressure streams.

Proof. This can be understood by examining the four-step operation of a WE, as depicted in Figure 3.

(ii) The volumetric flowrate of stream HP, i.e., \( V_{HP} \), must be less than that of stream LP, i.e., \( V_{LP} \). Thus, we have

\[
V_{HP} < V_{LP} \quad (17)
\]

Proof. Figure 6 illustrates a feasible stream match between streams HP and LP. The two arrowed lines representing these two streams intersect. The pressure differences at the high-pressure and low-pressure ends of the two streams, i.e., \( \Delta P_{h-e} \) and \( \Delta P_{l-e} \), respectively, are expressed as

\[
\Delta P_{h-e} = P_{HP}^s - P_{LP}^t \geq 0 \quad (18)
\]

Graphically, these two equations hold because the slope of the line representing stream LP, Slope LP, is less than that representing stream HP, Slope HP. Naturally, if Slope LP is equal to or greater than Slope HP, eqs 18 and 19 no longer hold.

Note that the slope of a line in the figure is the reciprocal of the volumetric flowrate of the corresponding stream. This is demonstrated below.

The amount of work energy transferred by a stream is expressed as

\[
W = V(P^t - P^s) = V\Delta P \quad (20)
\]

This equation can be rewritten as

\[
\frac{\Delta P}{W} = \frac{1}{V} \quad (21)
\]

Clearly, the ratio of \( \Delta P \) to \( W \), or the reciprocal of \( V \), is the slope of the line representing the corresponding stream.

(iii) The pressure difference at either the high-pressure or low-pressure end of the pair of streams matched, as defined in eqs 18 and 19, must be greater than the minimum pressure difference required, i.e.,

\[
\Delta P_{h-e} \geq \Delta P_{min} \quad (22)
\]

and

\[
\Delta P_{l-e} \geq \Delta P_{min} \quad (23)
\]

Proof. As stated previously, \( \Delta P_{min} \) represents the minimum requirement of the pressure difference at each side of a piston to be moved. Consequently, for the pressurization and depressurization processes, eqs 22 and 23 must be satisfied. The determination of \( \Delta P_{min} \) is an optimization problem; heuristically, \( \Delta P_{min} \) should be between 20 and 60 kg-cm/s (Fan et al., 1968).

6. Feasibility Analysis

A pair of streams satisfying the necessary and sufficient conditions for stream matching can be matched to exchange mechanical energy through a WE. This matching, however, can be accomplished in numerous ways. Hence, the identification of a feasible match region for each pair of streams is a necessary step toward a quick selection of the sequence of optimal stream matching.

Prior to conducting the feasibility analysis, let us define some additional terminology. The operating line for stream LP is termed line LP, and that for stream HP, line HP. In addition, the pressures of a stream into and out of a WE are termed inlet and outlet pressures, respectively. Note that the inlet pressure of the entering stream is often not equal to the source pressure of the same stream, and the outlet pressure of the leaving stream is frequently different from the target pressure of the same stream. This is due to the fact that, in general, the work duty of stream LP is not equal to that of stream HP.

A feasibility study on four cases, cases a–d, is delineated below.

Case a. Figure 7 provides a match situation in which the work energy available in stream HP is greater than
that required by stream LP. Thus, the work energy required for pressurizing stream LP can be entirely supplied through depressurizing stream HP.

By applying the necessary and sufficient conditions to this case, the infeasible regions for stream matching shaded in Figure 7 are identified. The feasible region is located between the two infeasible regions as indicated. In other words, no matter how the line representing stream LP is horizontally moved between the left and right extreme positions, these two streams can be feasibly matched.

(a-1) If line LP is located at the left extreme position, the inlet and outlet pressures of stream HP in a WTU are \( P_{HP}^s \) and \( P_{HP}^t \), respectively. This match gives rise to the generation of two segments of stream HP: stream HP\(^{1-1} \) with source pressure \( P_{HP}^s \) and target pressure \( P_{HP}^t \), and stream HP\(^{2-2} \) with source pressure \( P_{HP}^s \) and target pressure \( P_{HP}^t \).

(a-2) If line LP is located at the right extreme position, the inlet and outlet pressures of stream HP in a WTU are \( P_{HP}^s \) and \( P_{HP}^t \), respectively. This match generates two segments of stream HP: stream HP\(^{1-1} \) with source pressure \( P_{HP}^s \) and target pressure \( P_{HP}^t \), and stream HP\(^{2-2} \) with source pressure \( P_{HP}^s \) and target pressure \( P_{HP}^t \).

(a-3) If line LP is located somewhere between these two extreme positions, two segments of stream HP are generated: stream HP\(^{1-2} \) with source pressure \( P_{HP}^s \) and target pressure between \( P_{HP}^s \) and \( P_{HP}^t \), and stream HP\(^{2-1} \) with source pressure between \( P_{HP}^s \) and \( P_{HP}^t \) and target pressure \( P_{HP}^t \).

Case b. Figure 8 provides a match situation in which the work duty of stream HP is greater than that of stream LP as in case a. However, the work energy required for pressurizing stream LP may or may not be entirely supplied through depressurizing stream HP; this is different from case a. The infeasible regions for stream matching are shaded in the figure. The feasible region is located between the two infeasible regions as indicated.

(b-1) If line LP is located at the left extreme position, the inlet and outlet pressures of stream HP in a WTU are \( P_{HP}^s \) and \( P_{HP}^t \), respectively. The work required for pressurizing stream LP can be fully supplied through depressurizing stream HP. This match results in two segments of stream HP: stream HP\(^{1-1} \) with source pressure \( P_{HP}^s \) and target pressure \( P_{HP}^t \), and stream HP\(^{2-2} \) with source pressure \( P_{HP}^s \) and target pressure \( P_{HP}^t \).

(b-2) If line LP is located at the right extreme position, the inlet and outlet pressures of stream HP in a WTU are \( P_{HP}^s \) and \( P_{HP}^t \), respectively. The work energy exchanged through this match is less than the work energy required for pressurizing stream LP. After this match, a segment of stream HP and a segment of stream LP are formed. The former is termed stream HP\(^{1-1} \) with source pressure \( P_{HP}^s \) and target pressure \( P_{HP}^t \), and the latter is termed stream LP\(^{2-2} \) with source pressure \( P_{LP}^s \) and target pressure \( P_{LP}^t \).

(b-3) If line LP is located somewhere between the two extreme positions, the work required for pressurizing stream LP may or may not be fully supplied through depressurizing stream HP. The number and type of streams resulting from this match depend upon the position of line LP. In the feasible match region, there exists a critical position which differentiates the number and type of streams resulting from stream match and the amount of work energy exchanged.

(b-3-1) If line LP is located between the left extreme position and the critical position, the work energy required for pressurizing stream LP can be fully supplied through depressurizing stream HP. This match gives rise to two segments of stream HP as discussed in case b-1.

(b-3-2) If line LP is located between the critical position and the right extreme position, the work energy
required for pressurizing stream LP cannot be fully supplied through depressurizing stream HP. This match gives rise to a segment of stream HP and a segment of stream LP as discussed in case b-2.

b-3-3. If line LP is located at the critical position, the work energy required for pressurizing stream LP still can be fully supplied through depressurizing stream HP. This match gives rise to only one segment of stream HP with source pressure $P_{HP}^{s'}$ and target pressure $P_{HP}^{t}$.

Case c. Figure 9 presents a match situation in which the work energy available in stream HP is less than that required by stream LP. Thus, the work energy released through depressurizing stream HP can be entirely consumed in pressurizing stream LP. The infeasible regions for stream matching are shaded in the figure. The feasible region is located between the two infeasible regions as indicated.

c-1. If line HP is located at the left extreme position, the inlet and outlet pressures of stream LP in a WTU are $P_{LP}^{s}$ and $P_{LP}^{t}$, respectively. This match generates two segments of stream LP: stream LP"1 with source pressure $P_{LP}^{s}$ and target pressure $P_{LP}^{t}$, and stream LP"2 with source pressure $P_{LP}^{t}$ and target pressure $P_{LP}^{t}$.

c-2. If line HP is located at the right extreme position, the inlet and outlet pressures of stream LP in a WTU are $P_{LP}^{s}$ and $P_{LP}^{t}$, respectively. This match generates two segments of stream LP: stream LP"1 with source pressure $P_{LP}^{s}$ and target pressure $P_{LP}^{t}$, and stream LP"2 with source pressure $P_{LP}^{t}$ and target pressure $P_{LP}^{t}$.

c-3. If line HP is located somewhere between these two extreme positions, two segments of stream LP are generated: stream LP"1 with source pressure $P_{LP}^{s}$ and target pressure between $P_{LP}^{s}$ and $P_{LP}^{t}$, and stream LP"2 with source pressure between $P_{LP}^{t}$ and $P_{LP}^{t}$ and target pressure $P_{LP}^{t}$.

Case d. Figure 10 provides a match situation in which the work load of stream LP is less than that of stream HP as in case c. In this case, however, the work energy released through depressurizing stream HP may or may not be entirely consumed in pressurizing stream LP; this is different from case c. The infeasible regions for stream matching are shaded in the figure. The feasible region for stream matching is located between the two infeasible regions as indicated.

d-1. If line HP is located at the left extreme position, the inlet and outlet pressures of stream LP in a WTU are $P_{LP}^{s}$ and $P_{LP}^{t}$, respectively. The work energy exchanged through this match is less than the work energy available in stream HP. After this match, a segment of stream HP is generated; the former is stream HP"1 with source pressure $P_{HP}^{s'}$ and target pressure $P_{HP}^{t}$, and stream LP"2 with source pressure $P_{LP}^{t}$ and target pressure $P_{LP}^{t}$.

d-2. If line HP is located at the right extreme position, the inlet and outlet pressures of stream LP in a WTU are $P_{LP}^{s}$ and $P_{LP}^{t}$, respectively. The work energy released through depressurizing stream HP can be fully consumed in pressurizing stream LP. This match generates two segments of stream LP: stream LP"1 with source pressure $P_{LP}^{s}$ and target pressure $P_{LP}^{t}$, and stream LP"2 with source pressure $P_{LP}^{t}$ and target pressure $P_{LP}^{t}$.

d-3. If line HP is located somewhere between the two extreme positions, the work energy released through depressurizing stream HP may or may not be fully consumed in pressurizing stream LP. The number and type of streams resulting from this match depend upon the position of line HP. In the feasible match region, there exists a critical position which differentiates the number and type of streams resulting from a stream match and the amount of work exchanged.

d-3-1. If line HP is located between the left extreme position and the critical position, the work energy released through depressurizing stream HP cannot be fully consumed in pressurizing stream LP. This match gives rise to a segment of stream HP and a segment of stream LP as discussed in case d-1.
d-3. If line HP is located at the critical position, the work energy released through depressurizing stream HP can be fully consumed in pressurizing stream LP. This match gives rise to two segments of stream LP as discussed in case d-2.

d-3. If line HP is located at the critical position and the right extreme position, the work energy released through depressurizing stream HP can be fully consumed in pressurizing stream LP. This match gives rise to only one segment of stream LP with source pressure $P_{LP}^{s}$ and target pressure $P_{LP}^{t}$.

7. Heuristics for Stream Matching Selection

The exhaustive analysis on feasible stream matching in the preceding section reveals that numerous matching modes exist. Selection of an optimal match mode for each matching is the most difficult, yet the most essential aspect in WEN synthesis. In this section, heuristics are developed by taking into account all four cases of stream matching. These heuristics are developed to maximize the recovery of mechanical energy, to minimize the number of WE's to be employed, and to accelerate the synthesis process.

During the synthesis of an exchanger network, it is always desirable that the placement of each exchanger reduces rather than increases the size of the problem being solved. This general strategy has been interpreted as a heuristic rule, i.e., the stream elimination rule. This rule successfully applied to HEN and MEN syntheses is, in principle, also applicable to WEN synthesis. In WEN synthesis, however, neither of the two streams matched may be completely eliminated after matching under many circumstances, such as all four classes of cases except cases b-3-2 and d-3-2. Hence, we need to modify the stream elimination rule to the following.

Eliminate at least one of the two streams after matching them, if possible.

This rule should be adopted for any stream matching. In addition, other heuristic rules should be developed for all four cases as discussed in the preceding section.

Case a. Two segments of stream HP are always generated after stream HP and stream LP are matched (Figure 7). Note that the additional work lost by stream HP will not be influenced by the position of line LP. For instance, if stream LP is placed in the left extreme position, the total additional work lost by stream HP will be determined by the sum of $(P_{HP}^{c} - P_{LP}^{s})$ and $(P_{HP}^{t} - P_{HP}^{l})$. If stream LP is placed in the right extreme position, the total additional work lost by stream HP will be determined by the sum of $(P_{HP}^{c} - P_{LP}^{s})$ and $(P_{HP}^{t} - P_{HP}^{l})$; apparently, these two sums are equal. If line LP is located somewhere between the left and right extremes, the total additional work lost by stream HP will not be changed. If the remaining two segments of stream HP must be treated by conventional depressurizing methods, the operating cost will be changed if the input and output pressures of these two segments change. Nevertheless, in a WEN, the remaining two segments should be matched with other low pressure streams which will reduce the operating cost overall. In other words, the operating cost should be evaluated based on the whole process system. Thus, the selection of a match mode for this pair of streams only affects the source and target pressures of the resultant two high-pressure streams; this, in turn, affects any succeeding stream match between either of these two high-pressure streams and other low-pressure streams. The heuristic rule for this type of match is given below.

Select a match mode in the feasible match region so that the resultant two high-pressure streams can be matched completely with other unmatched low-pressure streams.

Case b. The feasible match region indicated in Figure 8 can be divided into two subregions: subregion 1 between the left extreme position and the critical position, and subregion 2 between the critical position and the right extreme position.

b-1. A match in subregion 1 will lead to two high-pressure streams generated from stream HP; this situation is the same as the one in case a.

b-2. A match in subregion 2 will lead to the generation of a high-pressure stream and a low-pressure stream from stream HP and stream LP, respectively. Each newly generated stream should be matched further with other streams to exchange work energy. If the work load of this newly generated stream is small, the use of a WE is not cost-effective. For the high-pressure stream, it can be depressurized through a valve, although its work energy is lost. For the low-pressure stream, a pump or compressor is required, thus increasing the cost of the system.

b-3. A match at the boundary of subregions 1 and 2 will lead to the generation of only one high-pressure stream, thereby reducing the size of the problem to be solved.

By comparing the above three match modes, we can develop the following heuristic rule.

Select the match mode at the critical position so that a low-pressure stream can be entirely eliminated and only one high-pressure stream is generated after the match.

Case c. In this case, two low-pressure streams are always generated from stream LP after it is matched with stream HP (Figure 9). The selection of match mode for a pair of streams only changes the source and target pressures of the resultant low-pressure streams. This, in turn, affects succeeding matches. The heuristic rule for this match is as follows:

Select a match mode in the feasible match region so that the resultant two low-pressure streams can be matched completely with other unmatched high-pressure streams.

Case d. The feasible match region indicated in Figure 10 can be divided into two subregions: subregion 1 between the left extreme position and the critical position, and subregion 2 between the critical position and the right extreme position.

d-1. A match in subregion 1 leads to a high-pressure stream and a low-pressure stream generated from streams HP and LP, respectively. Each newly generated stream is to be matched further with another stream to exchange work energy. If the work load of this newly generated stream is small, the use of a WE is not cost-effective. This situation is the same as that discussed in case b-2.

d-2. A match in subregion 2 leads to two new low-pressure streams generated from stream LP; this situation is the same as in case c.

d-3. A match at the boundary of subregions 1 and 2 leaves only one low-pressure stream unmatched, thereby reducing the size of the problem being solved.

By comparing the above three match modes, we can develop the following heuristic rule.
be determined at the preanalysis stage of process synthesis. As a result, the technology must be modified when it is applied to WEN synthesis.

9. Concluding Remarks

In this work, a novel concept of the work exchanger network (WEN) is introduced for the recovery of mechanical energy. An in-depth analysis of the task involved in synthesizing a WEN is provided. This has given rise to the derivation of the necessary and sufficient conditions for stream matching during the synthesis. The feasibility analysis of all possible stream matching modes results in the development of a number of heuristic rules, thereby facilitating stream matching. It has been recognized that the approaches for identifying a pinch point and for estimating the capital and operating costs used in HEN and MEN syntheses should be modified for WEN synthesis.

While the focus of this work is on the analysis of a WEN, the ultimate goal is to develop a systematic methodology for synthesizing a cost-effective WEN. The WEN synthesized will have great potential for enhancing the recovery of mechanical energy in the process and allied industries. More important, if a WEN is combined with a HEN and a MEN, thermal and mechanical energy as well as materials can be recovered simultaneously. Naturally, the economic implications of such a combination are highly significant.

Acknowledgment

This work has been supported (to Y. L. H.) by the NSF (CTS-9414494) and Wayne State University. It was also funded in part by the U.S. EPA under assistance agreement R-81953 to the Great Plains—Rocky Mountain Hazardous Substance Research Center for U.S. EPA Regions 7 and 8 with headquarters at Kansas State University. It has not been subjected to the Agency's peer and administrative review and, therefore, may not necessarily reflect the views of the Agency. No official endorsement should be inferred.

Nomenclature

\( C_i^r \) = source composition of the rich stream
\( C_i^c \) = target composition of the rich stream
\( C_i^M \) = source composition of the lean stream
\( C_i^L \) = target composition of the lean stream
\( LP \) = set of low-pressure streams
\( HP \) = set of high-pressure streams
\( M_{C_p_i} \) = normal heat capacity flow rate of the hot stream
\( M_{C_p_i} \) = normal heat capacity flow rate of the cold stream
\( M_i \) = normal mass flow rate of the rich stream
\( M_i \) = normal mass flow rate of the lean stream
\( M_{p} \) = mass load of the key component p of the rich stream
\( M_{p} \) = mass load of the key component p of the lean stream
\( N \) = total number of streams in a process
\( P_i^s \) = normal source pressure of stream i
\( P_i^t \) = normal target pressure of stream i
\( Q_i \) = heat load of the hot stream
\( Q_i \) = heat load of the cold stream
\( S_i \) = stream i
\( T_i^h \) = source temperature of the hot stream
\( T_i^h \) = target temperature of the hot stream
\( T_i^c \) = source temperature of the cold stream
\( T_i^c \) = target temperature of the cold stream
\( V_i \) = normal volumetric flow rate of stream i
\( W_i \) = normal work energy of stream i
\( \Delta P = \) pressure difference between the source and target of a stream
\( \Delta P_{h-e} = \) pressure difference at the high-pressure ends of a pair of streams
\( \Delta P_{l-e} = \) pressure difference at the low-pressure ends of a pair of streams
\( \Delta P_{\text{min}} = \) minimum allowable pressure difference between the two sides of the piston in a work exchanger

Superscripts
\( s = \) source
\( t = \) target

Subscripts
\( C = \) cold stream
\( i = \) stream \( i \)
\( j = \) stream \( j \)
\( H = \) hot stream
\( HP = \) high-pressure stream
\( L = \) lean stream
\( LP = \) low-pressure stream
\( N = \) total number of streams in a process
\( p = \) key component
\( R = \) rich stream

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Received for review December 6, 1995
Revised manuscript received June 12, 1996
Accepted June 14, 1996

IE9507383

*Abstract published in Advance ACS Abstracts, September 1, 1996.*